Math 150 Plane Trigonometry
Professor John Maginnis

Midterm Exam 1
February 2, 2012

Your name: ______________________________

Rec. Instr.: ______________________________

Rec. Time: ______________________________

Instructions:

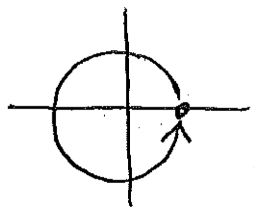
Show all your work in the space provided under each question. Please write neatly and present your answers in an organized way. You may use your one sheet of notes, and also a calculator. This exam is worth 60 points.

The chart below indicates how many points each problem is worth.

Problem	1	2	3	4	5
Points	/8	/6	/6	/6	/6
Problem	6	7	8	9	10
Points	/6	/6	/6	/4	/6

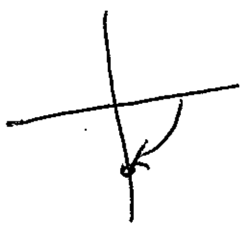
1. Find the exact values of the following. Explain how you found your answer.

(a) $\cos(360°)$ $= \boxed{1}$ 360° is a complete circle.



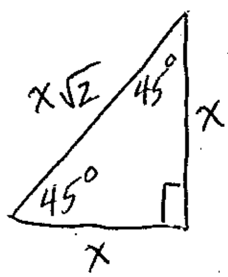
The point on the unit circle is $(1,0)$.

(b) $\sin(-\frac{\pi}{2}$ radians$)$ $= \boxed{-1}$ $\frac{\pi}{2}$ radians $= 90°$



The point on the unit circle is $(0,-1)$.

(c) $\tan(45°)$ $= \boxed{1}$

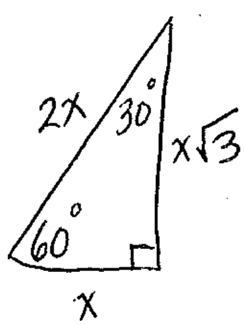


This triangle is isoceles with two equal sides.

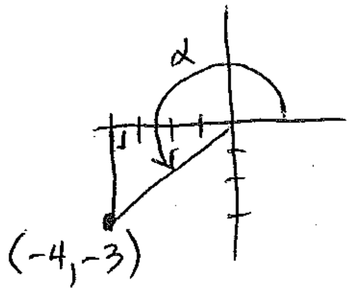
$\tan(45°) = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{x} = 1.$

(d) $\sec(\frac{\pi}{3}$ radians$)$ $= \frac{1}{\cos(\frac{\pi}{3})} = \frac{1}{1/2} = \boxed{2}$

$\frac{\pi}{3}$ radians $= 60°$



2. The point $(-4, -3)$ lies on the terminal side of an angle α. Find the exact values of all six trigonometric functions.



Note α lies in the third quadrant.

$$\sin(\alpha) = -3/5$$

$$\cos(\alpha) = -4/5$$

$$\tan(\alpha) = \frac{-3}{-4} = \frac{3}{4}$$

$$\sec(\alpha) = \frac{1}{\cos(\alpha)} = -5/4$$

$$\csc(\alpha) = \frac{1}{\sin(\alpha)} = -5/3$$

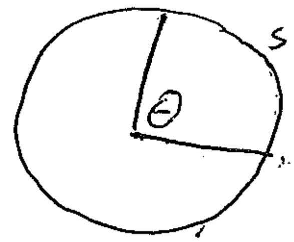
$$\cot(\alpha) = \frac{1}{\tan(\alpha)} = 4/3$$

Note $(-4)^2 + (-3)^2 = 16 + 9 = 25 = 5^2$

so the hypotenuse has length 5.

The point on the unit circle is $\left(\frac{-4}{5}, \frac{-3}{5}\right)$.

3. Find the central angle of the sector of a circle which has an arc length of 6 meters and an area of 9 square meters.



If the central angle θ is in radians, then $s = R\theta$ and $A = \frac{1}{2}R^2\theta$ where R is the radius of the circle.

Thus $6 = R\theta$ and $9 = \frac{1}{2}R^2\theta$, $18 = R^2\theta$.

Then $\frac{18}{6} = \frac{R^2\theta}{R\theta} = R$, so the radius is $R = 3$ meters.

Then $6 = R\theta = 3\theta$, so $\theta = \frac{6}{3} = \boxed{2 \text{ radians}}$.

(Note $\theta = \frac{360}{\pi} \approx 114.59$ degrees.)

4. (a) Convert $\alpha = 3$ radians to degrees, minutes, and seconds, rounding to the nearest second.

$$\alpha = 3 \times \frac{180}{\pi} = \frac{540}{\pi} \approx 171.8873385 \text{ degrees}$$

$$.8873385 \times 60 = 53.24031236 \text{ minutes}$$

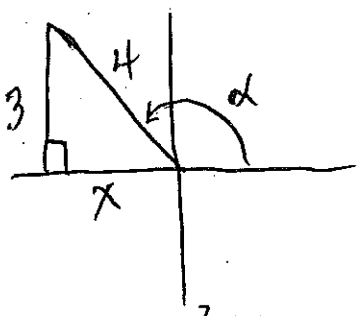
$$.24031236 \times 60 = 14.4187413 \text{ seconds} \quad \boxed{\alpha = 171^\circ 53' 14''}$$

(b) Convert $\beta = 72^\circ$ to radians (give the exact value).

$$\beta = 72 \times \frac{\pi}{180} = \frac{72\pi}{180} = \boxed{\frac{2\pi}{5} \text{ radians}}$$

5. If $\sin(\alpha) = \frac{3}{4}$ and $\cot(\alpha) < 0$, find the exact values of all six trigonometric functions.

Since $\cot(\alpha) = \dfrac{\cos(\alpha)}{\sin(\alpha)} < 0$ and $\sin(\alpha) > 0$, we must have $\cos(\alpha) < 0$. Thus α lies in quadrant two.



Pythagorean Theorem $x^2 + 9 = 16$

$$x^2 = 7$$

$$x = -\sqrt{7}$$

or use $\sin^2(\alpha) + \cos^2(\alpha) = 1$

$$\frac{9}{16} + \cos^2(\alpha) = 1$$

$$\cos^2(\alpha) = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos(\alpha) = -\sqrt{\frac{7}{16}} = \frac{-\sqrt{7}}{4}$$

$$\boxed{\begin{array}{l} \sin(\alpha) = \frac{3}{4}, \quad \csc(\alpha) = \frac{4}{3} \\ \cos(\alpha) = \frac{-\sqrt{7}}{4}, \quad \sec(\alpha) = \frac{-4}{\sqrt{7}} \\ \qquad\qquad\qquad \sec(\alpha) = \frac{-4\sqrt{7}}{7} \\ \tan(\alpha) = \frac{3}{-\sqrt{7}} = \frac{-3\sqrt{7}}{7}, \quad \cot(\alpha) = \frac{-\sqrt{7}}{3} \end{array}}$$

6. Verify the identity $\csc(\alpha) - \sin(\alpha) = \cos(\alpha)\cot(\alpha)$.

$$\frac{1}{\sin(\alpha)} - \sin(\alpha) = \cos(\alpha) \cdot \frac{\cos(\alpha)}{\sin(\alpha)}$$

$$\frac{1 - \sin^2(\alpha)}{\sin(\alpha)} = \frac{\cos^2(\alpha)}{\sin(\alpha)}$$

$$1 - \sin^2(\alpha) = \cos^2(\alpha)$$

$$1 = \sin^2(\alpha) + \cos^2(\alpha)$$

(Pythagorean Theorem)

7. Give one positive and one negative coterminal angle for each of the following.

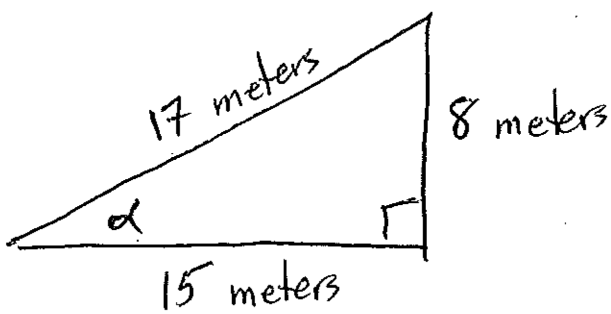
(a) 2 degrees

$$2 + 360 = \boxed{362^\circ}$$
$$2 - 360 = \boxed{-358^\circ}$$

(b) 2 radians

$$\boxed{\begin{array}{l} 2 + 2\pi \text{ radians} \\ 2 - 2\pi \text{ radians} \end{array}}$$

8. A right triangle has three sides of lengths 8 meters, 15 meters, and 17 meters. Note that $(8)^2 + (15)^2 = (17)^2$. The angle α in the triangle is opposite the side of length 8 meters. Find the exact values of all six trigonometric functions.



$$\sin(\alpha) = 8/17$$

$$\cos(\alpha) = 15/17$$

$$\tan(\alpha) = 8/15$$

$$\csc(\alpha) = \frac{1}{\sin(\alpha)} = 17/8$$

$$\sec(\alpha) = \frac{1}{\cos(\alpha)} = 17/15$$

$$\cot(\alpha) = \frac{1}{\tan(\alpha)} = 15/8$$

9. (a) Find the angle that is complementary to one radian.

$\boxed{\frac{\pi}{2} - 1}$ radians

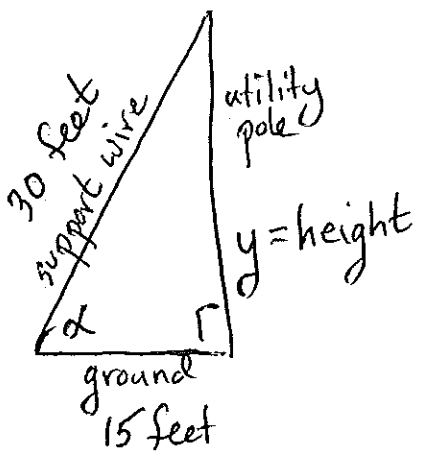
(b) Find the angle that is supplementary to 107°23′.

$180^\circ - 107^\circ 23'$

$$\begin{array}{r} 179^\circ 60' \\ -\,107^\circ 23' \\ \hline 72^\circ 37' \end{array}$$

$\boxed{72^\circ 37'}$

10. A support wire of length 30 feet is attached to the top of a vertical utility pole and to the ground at a point 15 feet from the base of the pole. Find the height of the utility pole and the angle at which the support wire meets the ground.



$\cos(\alpha) = \frac{15}{30} = \frac{1}{2}$, so $\boxed{\alpha = 60^\circ}$

$\sin(\alpha) = \sin(60^\circ) = \frac{y}{30}$, so

$y = 30\sin(60^\circ) = 30\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{y = 15\sqrt{3} \text{ feet}} \approx 25.98$ feet

or use the Pythagorean Theorem

$15^2 + y^2 = 30^2$

$225 + y^2 = 900$

$y^2 = 900 - 225 = 675$

$y = \sqrt{675}$ feet

Note reflecting the triangle yields an equilateral triangle with all three sides of length 30 feet. Thus the three angles are equal, and so $\alpha = \frac{180^\circ}{3} = 60^\circ$.